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Some Remarks on the Dynamical Origin of Charge and Space-Time Quantisation

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Abstract

It is argued here that the concept of dynamical origin of charge as formulated in a previous paper requires the quantisation of space-time. Indeed, in this scheme, it is pointed out that the quantisation of electric charge in unit of e is a direct consequence of this space-time quantisation.

In a recent paper (Bandyopadhyay, 1973), we have developed a model of leptons on the basis of the non-local field theory where we have taken that the mass, as well as the charge of the electron and muon, is of dynamical origin. This procedure helps us to unify weak and electromagnetic interaction and the accompanying violation of symmetry generates the photon as a Goldstone boson. In this model, the electron and muon are depicted as $(\nu_e s)$ and $(\nu_\mu s)$ respectively, where s represents the system of photons interacting 'weakly' at n space-time points with the extended structure of a two-component neutrino. The two other components corresponding to the positive and negative energy states are formed when the form factor associated with the interaction changes its sign implying that particles and anti-particles are mirror reflections of each other (Bandyopadhyay, 1963). Evidently, this formalism is in conformity with the concept of CP invariance.

It may be mentioned here that the CPT theorem, which is the direct product of Lorentz invariance and local field theory, suggests that 'charge conjugation invariance' which, in a sense, states that to every particle there exists an anti-particle of opposite charge and the same mass, should be related with oridinary space-time transformations. But this becomes possible only when a space-time description of electric charge can be achieved. The model of leptons developed in I, however, gives us such a space-time description of charge. In this note we want to show that such a space-time description of electric charge also suggests the quantisation of space-time. Indeed, to make the description meaningful, it is necessary to introduce the concept of elementary space-time domain and the universal 'space-time continuum' is to be described as a collection of all such elementary domains.

For our convenience, we here recapitulate the main arguments that lead to a geometrical description of electric charge (Bandyopadhyay, 1973). To obtain this, we must use non-local fields for the interacting particles.

Let us consider the two-component spinor wave-function $\psi(x, r)$ where x and r are external and internal space-time variables. It is considered that $\psi(x, r)$ satisfies the relation

$$\psi(x) = \int d^4r\,\psi(x, r) \tag{1}$$

It is further contended that the non-local spinor $\psi(x, r)$ obeys the Dirac equation in terms of the variable x

$$\left(\gamma^\mu \frac{\partial}{\partial x_\mu} + \mu\right)\psi(x, r) = 0 \tag{2}$$

The spinor current is expressed as

$$c^\mu(x) = \int d^4r\, d^4s\, \bar{\psi}(x, r)\gamma^\mu \psi(x, s) \tag{3}$$

Now assuming that the electromagnetic field quantity $A_\mu(Y, t)$ also satisfies a similar relation

$$A_\mu(Y) = \int d^4t A_\mu(Y, t) \tag{4}$$

we take n-photon fields at different space-time points in thie external space as follows.

$$\begin{aligned} &A_\mu(Y - \tfrac{1}{2}t_1) + A_\mu(Y + \tfrac{1}{2}t_1) + A_\mu(Y - \tfrac{1}{2}t_2) + A_\mu(Y + \tfrac{1}{2}t_2) + \cdots \\ &\qquad + A_\mu(Y - \tfrac{1}{2}t_m) + A_\mu(Y + \tfrac{1}{2}t_m) \end{aligned} \tag{5}$$

From this, we see that when $t \to 0$ the expression just reduces to the single point potential given by $nA_\mu(Y)$ when $n = 2m$. Thus the interaction Lagrangian for n-photon weak interactions with the spinor takes the form

$$\begin{aligned} L_{\mathrm{I}} &= ig\left[\sum_{i=1}^{m} c^\mu(x)A_\mu(Y - \tfrac{1}{2}t_i) + \sum_{i=1}^{m} c^\mu(x)A_\mu(Y + \tfrac{1}{2}t_i)\right] \\ &= ig \sum_{i=1}^{n} \int d^4r\, d^4s\, d^4t\, [\bar{\psi}(x, r)\gamma^\mu \psi(x, s)A_\mu(Y_i, t) \\ &\quad + \bar{\psi}(x, r)\gamma^\mu \psi(x, s)A_\mu(\bar{Y}_i, t)] \end{aligned} \tag{6}$$

where $Y_i = Y - \frac{1}{2}t_i$, $\bar{Y}_i = Y + \frac{1}{2}t_i$ and g is the dimensionless weak coupling constant which is taken to have the value $g = 10^{-10}e$ (Bandyopadhyay, 1968).

Now taking m such that $(e/2m) = g$, the weak coupling constant, we note that the system of interactions (6) in the limit $t \to 0$ just reduces to the formal electromagnetic coupling, i.e.

$$\int d^4r\, d^4s\, d^4t \bar{\psi}(x, r)\gamma^\mu \psi(x, s) A_\mu(Y, t) \tag{7}$$

Thus in the limit $t \to 0$, n-photon weak interactions can be considered to be 'equivalent' to the proper electromagnetic interaction (7) and by this a geometrical description of e in terms of g is obtained.

To show that the coupling constant e obtained in such manner actually represents the 'charge', we have shown in I, that equation (6) can give rise to a less symmetric solution which generates the 'electromagnetic' interaction from a system of n-photon 'weak' interactions at different space-time points. This violation of symmetry occurs due to the fact that (1) interaction (7) is equivalent to an interaction involving local fields with form factor which gives mass to the bare spinor and (2) the positive and negative sign of the form factor is found to correspond to the positive and negative energy states and thus a four-component spinor can be formed from a bare two-component spinor. The generation of mass through the interaction violates the symmetry corresponding to the invariance under the transformation $\psi \to e^{i\alpha\gamma_5}\psi$ inherent in the original system of 'weak' interactions involving the bare two-component spinor as given by equation (6) and thus, in this scheme, electromagnetic interaction is generated through the spontaneous breakdown of symmetry and the photon appears as a Goldstone boson.

A direct consequence of this interpretation of charge is that the generation of charge is associated with the generation of mass and, as such, there cannot be any massless charged particle. Also we note that the number n of the system of interactions in equation (6) bears a very crucial sense. For n must be a unique number otherwise we could get any amount of charge and mass of a lepton formed in this manner. Since we know that all charges in nature occur in units of e (provided we accept there is no fractionally charged particle like quarks), n must be specified by the quantity e/g, where g is the photon-neutrino weak coupling constant.

Now, the point is, how we can specify this number n in nature? It is presumed here that space-time in nature is quantised such that the whole 'space-time continuum' is considered as a collection of elementary space-time domains. Each such domain is specified by the fact that no physical measurement is possible within this. This elementary domain can be considered to be the 'seat' of an elementary particle like the electron and muon. *The number n of interactions as specified by the ratio e/g is determined by the dimension of this elementary domain such that no more photon can be accommodated there.* Thus we can get the unique charge e for a lepton. This analysis, of course, requires the introduction of a fundamental length l_0 in nature as a measure of the dimension of this elementary domain. It may be added that the necessity of such a fundamental length has also been emphasised by other physicists in other contexts (Yukawa, 1965). In fact, Heisenberg stated that if we have three fundamental units, c, h and l_0, our natural system of units is complete. Yukawa also stressed the necessity of a fundamental length in his attempt for a space-time description of elementary particles. Of course, we must mention that the fundamental length considered here need not necessarily be identical with the fundamental unit of length which can be constructed from h, c and the gravitational constant. Indeed, the value of l_0 derived from this is very small ($\simeq 10^{-33}$ cm). Whatever the value of l_0 we can determine, the main point of emphasis here is that the microspace-time manifests itself as discrete domains and not as a continuous medium. Thus we find that the concept of dynamical origin of charge and the quantisation of charge in units of e is closely associated with the concept of space-time quantisation and the introduction of a fundamental length in nature.

Finally, we may remark here that if n is thus specified and all charges in nature are of dynamical origin, then this formalism forbids the existence of fractionally charged particles as envisaged in the quark model of hadrons. Indeed, in a recent paper (Bandyopadhyay, 1973), we have shown that hadrons can indeed be considered to be composied of leptons and the internal quantum numbers such as isospin, strangeness and baryon number, as well as the classification and mass spectrum of hadrons, can be interpreted from the very configurational characteristics of hadrons. Thus the necessity of the introduction of fractionally charged particles to interpret the spectrum of hadrons is avoided and this explains why quarks have not yet been observed.

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